B 1. Solve each of the following equations exactly.

(a) $2^x = 5$
(b) $3^x = 10$
(c) $10^{x-4} = 7$
(d) $5^{1-x} = 2$
(e) $4^{2x} = 15$
(f) $6^{\frac{x}{3}} = 29$

2. Find the roots of the following equations correct to 4 decimal places.

(a) $10^x = 16$
(b) $7^x = 43$
(c) $2^{-x} = 6$
(d) $3^{1+x} = 36$
(e) $4^{3x} = 21$
(f) $8^{-\frac{x}{3}} = 20$
(g) $5^{2x+3} = 30$
(h) $2^{x^2} = 10$

3. Solve for x.

(a) $\log_2 x = \log_2 5 + \log_2 3$
(b) $\log_2 x = \log_2 18 - \log_2 6$
(c) $\log_{10} x + \log_{10} 12 = \log_{10} 8$
(d) $\log_{10} x = 1 + \log_{10} 2$
(e) $\log_3 x + \log_3 (x - 1) = \log_3 (2x)$
(f) $\log_9 (x - 5) + \log_9 (x + 3) = 1$
(g) $\log_2 (x + 1) - \log_2 (x - 1) = 1$
(h) $3 \log_2 x = \log_2 8$
(i) $\log_{10} x = 3 \log_{10} 7$
(j) $4 \log_6 x = \log_6 625$

C 4. Solve for x.

(a) $\log_2 (3x + 2) - \log_2 (x - 2) = 3$
(b) $\log_{10} (1 + \sqrt{x}) = 1 + \log_{10} (1 - \sqrt{x})$

A 1. Express as sums or differences of logarithms.

(a) $\log_{10} (8 \times 13)$
(b) $\log_2 (9.1 \times 6.3)$
(c) $\log_5 (14 \times 8.1)$
(d) $\log_5 \left(\frac{11}{37}\right)$
(e) $\log_8 \left(\frac{104}{97.2}\right)$
(f) $\log_{10} \left(\frac{2}{\pi}\right)$
(g) $\log_3 2\pi$
(h) $\log_2 (19 \times 97)$
(i) $\log_{12} (16 \div 65)$
(j) $\log_{10} xy$
(k) $\log_{10} \frac{x}{y}$
(l) $\log_x (AB)$

2. Express as logarithms of products or quotients.

(a) $\log_{10} 89 + \log_{10} 14$
(b) $\log_5 12.2 + \log_5 2.79$
(c) $\log_2 75 - \log_2 36$
(d) $\log_{10} 634 - \log_3 149$
(e) $\log_6 2 + \log_6 9$
(f) $\log_7 54 - \log_7 9$
(g) $\log_{10} x + \log_{10} y$
(h) $\log_2 x - \log_2 y$
(i) $\log_{10} 36 - \log_{10} 4$
(j) $\log_9 12 + \log_9 5$

3. Apply the Power Law to the following.

(a) $\log_{10} 68^2$
(b) $\log_2 3.9^5$
(c) $\log_5 \pi^{10}$
(d) $\log_{10} 7^{\frac{3}{4}}$
(e) $\log_3 5^{\frac{1}{2}}$
(f) $\log_5 \sqrt{3}$
(g) $\log_{10} 8^{-1}$
(h) $\log_{10} \left(\frac{1}{12}\right)$
(i) $\log_{10} x^9$
(j) $2 \log_{10} 37$
(k) $8 \log_2 21$
(l) $3 \log_5 2$
(m) $\frac{1}{3} \log_5 97$
(n) $\frac{1}{2} \log_{10} 9$
(o) $-\log_{10} 5$
(p) $-\frac{1}{2} \log_{10} 16$
(q) $\log_2 x^y$
(r) $m \log_6 A$

4. Apply the Laws of Logarithms to the following.

(a) $\log_{12} (82 \times 28)$
(b) $\log_2 (9 \times 13 \times 14)$
(c) $\log_5 9^{20}$
(d) $\log_3 (79 \div 53)$
(e) $2 \log_{10} 6$
(f) $\log_2 (LMN)$
(g) $\frac{1}{2} \log_{10} 49$
(h) $\log_2 \left(\frac{937}{1005}\right)$
(i) $\log_{10} \left(\frac{1}{67}\right)$
(j) $\log_5 \sqrt{83}$
(k) $\log_a (5x)$
(l) $-\log_3 8$
(m) $\log_2 6 + \log_2 7$
(n) $\log_{10} 28 - \log_{10} 4$

B 5. Given the approximate values
$\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, and $\log_{10} 5 = 0.6990$, evaluate the following.

(a) $\log_{10} 6$
(b) $\log_{10} 15$
(c) $\log_{10} 4$
(d) $\log_{10} 18$
(e) $\log_{10} 125$
(f) $\log_{10} \left(\frac{5}{2}\right)$
(g) $\log_{10} 1.5$
(h) $\log_{10} \left(\frac{3}{5}\right)$
(i) $\log_{10} 7.5$
(j) $\log_{10} \sqrt{5}$
(k) $\log_{10} \sqrt{3}$
(l) $\log_{10} \frac{1}{2}$
(m) $\log_{10} 200$
(n) $\log_{10} 50\,000$
(o) $\log_{10} 0.003$
(p) $\log_{10} \sqrt{54}$

6. Use the Laws of Logarithms to evaluate the following.

(a) $\log_4 2 + \log_4 32$
(b) $\log_{10} 1.25 + \log_{10} 80$
(c) $\log_3 108 - \log_3 4$
(d) $\log_2 80 - \log_2 5$
(e) $\log_{12} 16 + \log_{12} 9$
(f) $\log_3 \sqrt[3]{9}$
(g) $\log_2 8^{27}$
(h) $\log_{10} \sqrt{0.1}$
(i) $\log_8 6 - \log_8 3 + \log_8 2$
(j) $\log_5 5\sqrt{5}$

8. Express as a single logarithm.

(a) $\log_3 6 + 4 \log_3 2$
(b) $\log_6 3 + \frac{1}{2} \log_6 5 - \log_6 2$
(c) $\log_2 a + \log_2 b - \log_2 c$
(d) $\log_{10} a + \frac{1}{2} \log_{10} b - 2 \log_{10} c$
(e) $\frac{1}{2}[\log_{10} x + \log_{10} y] - 2 \log_{10} c$
(f) $\frac{1}{2}[\log_5 a + 2 \log_5 b) - 3 \log_5 c]$
(g) $\log_2(a + b) + \log_2 (a - b) - 2 \log_2 a$
(h) $\log_2 a + b \log_2 c - d \log_2 e$

Solutions — Part A

1. (a) $\log_{10}8 + \log_{10}13$ (b) $\log_2 9.1 + \log_2 6.3$ (c) $\log_5 14 + \log_5 8.1$
   (d) $\log_5 11 - \log_5 37$ (e) $\log_8 104 - \log_8 97.2$ (f) $\log_{10}2 - \log_{10}\pi$
   (g) $\log_3 2 + \log_3 \pi$ (h) $\log_2 19 + \log_2 97$ (i) $\log_{12}16 - \log_{12}65$
   (j) $\log_{10}x + \log_{10}y$ (k) $\log_{10}x - \log_{10}y$ (l) $\log_a A + \log_a B$

2. (a) $\log_{10}(89 \times 14)$ (b) $\log_5(12.2 \times 2.79)$ (c) $\log_2(\frac{75}{36})$

   (d) $\log_3(\frac{634}{149})$ (e) $\log_6(18)$ (f) $\log_7(6)$

   (g) $\log_{10}(xy)$ (h) $\log_2\left(\frac{x}{y}\right)$ (i) $\log_{10}(9)$

   (j) $\log_9(60)$

3. (a) $2\log_{10}68$ (b) $5\log_2 3.9$ (c) $10\log_5 \pi$
   (d) $\frac{3}{4}\log_{10}7$ (e) $\frac{1}{2}\log_3 5$ (f) $\frac{1}{2}\log_5 3$
   (g) $-\log_{10}8$ (h) $-\log_{10}12$ (i) $9\log_{10}x$
   (j) $\log_{10}37^2$ (k) $\log_5 21^6$ (l) $\log_5 2^3$
   (m) $\log_5 97^{\frac{1}{3}}$ (n) $\log_{10}3$ (o) $\log_{10}5$

4. (a) $\log_{12}82 + \log_{12}28$ (b) $\log_2 9 + \log_2 13 + \log_2 14$ (c) $20\log_5 9$
   (d) $\log_3 79 - \log_3 53$ (e) $\log_{10}36$ (f) $\log_2 L + \log_2 M + \log_2 N$
   (g) $\log_{10}7$ (h) $\log_2 937 - \log_2 1005$ (i) $-\log_{10}67$
   (j) $\frac{1}{2}\log_5 83$ (k) $\log_a 5 + \log_a x$ (l) $\log_3(\frac{1}{8})$
   (m) $\log_2 42$ (n) $\log_{10}7$

5. (a) 0.7781   (b) 1.1761   (c) 0.6020   (d) 1.2552
   (e) 2.097   (f) 0.398   (g) 0.1761   (h) -0.2219
   (i) 0.8751   (j) 0.3495   (k) 0.119275   (l) -0.3010
   (m) 2.301   (n) 4.699   (o) -2.5229 (p) 0.8662

6. (a) 3   (b) 2   (c) 3   (d) 4   (e) 2
   (f) $\frac{2}{3}$ (g) $\frac{2}{3}$ (h) 81   (i) $-\frac{1}{2}$ (j) $\frac{3}{2}$

8. (a) $\log_3(6 \times 2^4)$ (b) $\log_6\left(\frac{3 \times \sqrt{5}}{2}\right)$ (c) $\log_2\left(\frac{ab}{c}\right)$ (d) $\log_{10}\left(\frac{a\sqrt{b}}{c^2}\right)$

   (e) $\log_{10}\left(\frac{\sqrt{xy}}{c^2}\right)$ (f) $\log_5\left(\sqrt{\frac{ab^2}{c^3}}\right)$ (g) $\log_2\left(\frac{a^2-b^2}{a^2}\right)$ (h) $\log_2\left(\frac{ac^b}{e^d}\right)$

Solutions — Part B

1. (a) $x = \log_2 5$ (b) $x = \log_3 10$ (c) $x = \log_{10}7 + 4$
   (d) $x = 1 - \log_5 2$ (e) $x = \frac{1}{2}\log_4 15$ (f) $x = 3\log_6 29$

2. (a) 1.2041   (b) 1.9328   (c) -2.5849 (d) 2.2618
   (e) 0.7320   (f) -4.3219 (g) -0.4433 (h) ± 1.8226

3. (a) $x = 15$ (b) 3   (c) $x = \frac{2}{3}$ (c) $x = 20$ (e) $x = 3$
   (f) $x = 6$ (g) 3   (h) $x = 2$ (i) $x = 7^3 = 343$ (j) $x = 5$

4. (a) $x = \frac{18}{5}$ (b) $x = \frac{81}{121}$