## The Sine Law



| The Low of Sines |  |
| :---: | :---: |
| $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ | $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ |
| Use to find ANGLES | Use to find sides |

Example 1 Solve the following triangle.


Example 2 Find the measure of angle $x$.


## The Sine Law and the Ambiguous Case

Suppose you were asked to solve $\triangle A B C$, where $<A=36^{\circ}, a=5.9 \mathrm{~cm}$, and $b=7.8$ cm . A good starting point would be to draw the triangle but when doing so, we need to be careful because sometimes there are 2 triangles with the given properties.

We must provide both solutions:

This is called the ambiguous case. Given $\Varangle A, \mathrm{a}$ and b , if $\mathbf{a}<\mathbf{b}$ there are 3 possibilities:
$\boldsymbol{a}<\boldsymbol{b} \sin A$ means no triangle possible (no solution)
$\boldsymbol{a}=\boldsymbol{b} \sin A$ means 1 right angle triangle is possible (1 solution)
$\boldsymbol{a}>\boldsymbol{b} \sin \boldsymbol{A}$ means $\mathbf{2}$ triangles are possible (2 solutions)

Example 3 Determine the number of solutions. Then, determine the length of the third side.
a) In $\triangle A B C$ where $\Varangle A=30^{\circ}, \mathrm{a}=3 \mathrm{~cm}$ and $\mathrm{c}=5 \mathrm{~cm}$.
b) In $\triangle P Q R, \angle P=49^{\circ}, p=12.2 \mathrm{~cm}$, and $q=18 \mathrm{~cm}$.
c) In $\triangle K L M, \angle K=36^{\circ}, k=5.0 \mathrm{~m}$, and $I=8.5 \mathrm{~m}$.

