# Stretches, Compressions, and Reflections

For the equation $y = a\sin x$ and $y = a\cos x$, $|a|$ is the **amplitude**, which is the distance between the horizontal axis and the maximum or the horizontal axis and the minimum.

When: **0 < a < 1**, the function is **compressed vertically by a factor of a**.
**a > 1**, the function is **stretched vertically by a factor of a**.
**a < 0**, the function is **reflected in the x axis**.

For the equation $y = \sin kx$ and $y = \cos kx$, **k** tells us the number of cycles that will occur in $360^o$ (normal length of one cycle). **k** changes the **period length** which is the length of one cycle and is calculated using the formula, $\boldsymbol{period} = \frac{\mathbf{360°}}{|\boldsymbol{k}|}$.

When: **0 < k < 1**, the function is **stretched horizontally by a factor of $\frac{\mathbf{1}}{\boldsymbol{k}}$.**
**k > 1**, the function is **compressed horizontally by a factor of $\frac{\mathbf{1}}{\boldsymbol{k}}$.**
**k < 0**, the function is **reflected in the y axis.**

**Example** Determine the vertical stretch or compression, horizontal stretch or compression, reflections, amplitude, and period for each function.

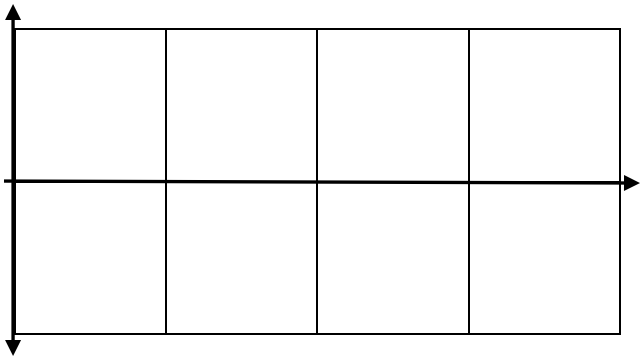
a. $y = 4\cos 3x$

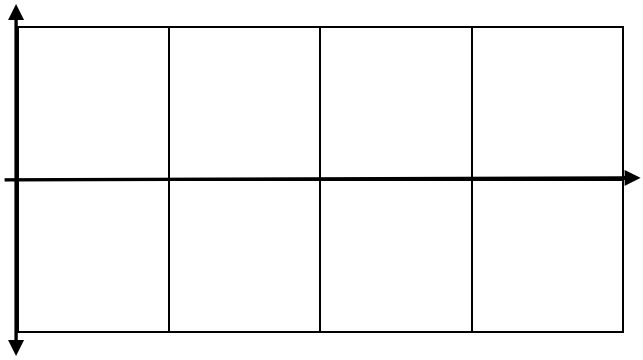
b. $y = -2\,sin\,\frac{1}{4}x$

c. $y = \frac{2}{3}\cos(-4x)$

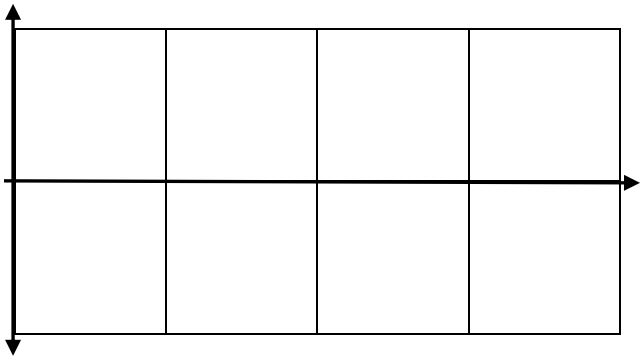
We will now look at graphing sinusoidal functions which have undergone stretches, compressions, and/or vertical reflections.

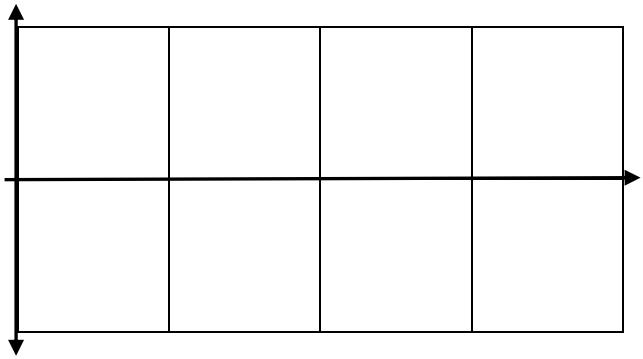
Recall that the graphs of y=sin x and y = cos x are very similar. The first cycle of each graph looks like:





**If a is negative**, the first cycle of each graph looks like:



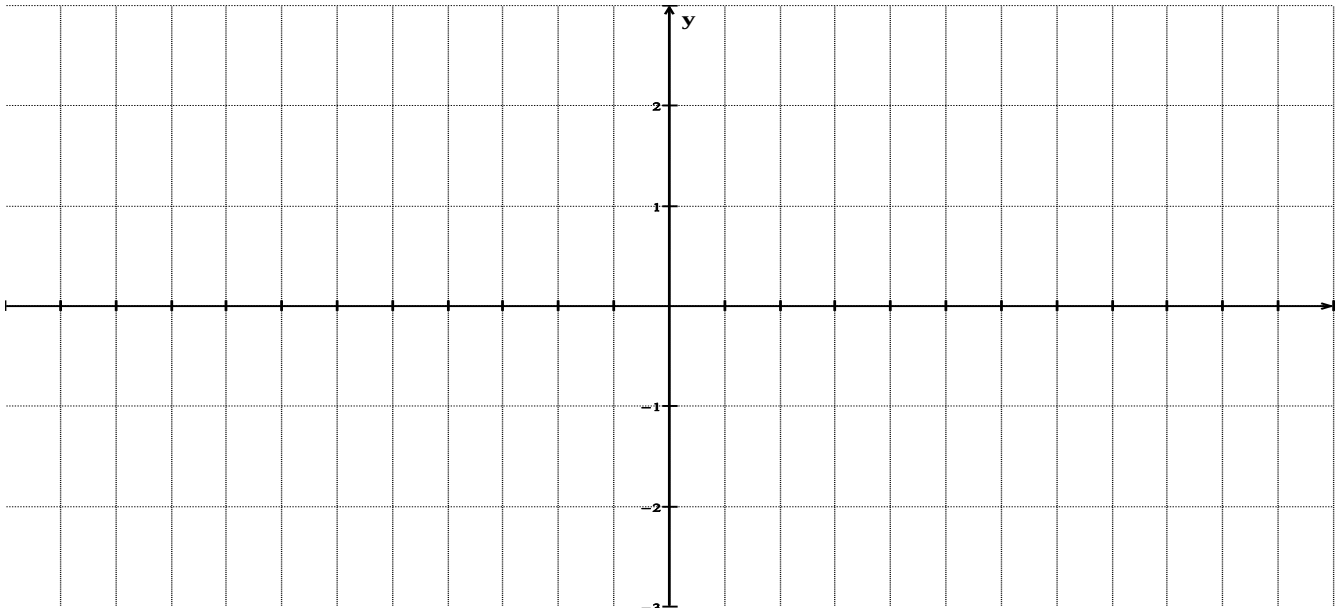


We will graph transformed trig functions by applying the transformations to the first cycle of the graph first using the box method. Therefore, you will probably want to refer back to these 4 sketches until you get used to the patterns in each graph.

**Introduction to Graphing using the "Box" Method**

1. Use the **amplitude** to determine the **maximum and minimum** values. (**a** value)
2. Determine the **period** to calculate the right boundary of one cycle. (**k** value)
3. Sketch the first cycle, paying attention to whether there are reflections to deal with (see below)

**Example 1** Graph $y = 3\cos\frac{1}{2}x$




**Example 2** Graph $y = -2\sin 3x$.