Properties of Polynomial Functions

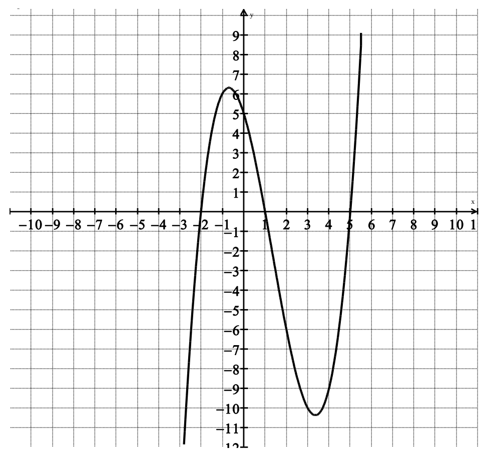
A. Graphing from Factored Form

Recall: a) Roots = Zeros = x-intercepts

b) The parabola with the equation $y = -2(x+4)(x-3)$ has the x-intercepts -4, 3

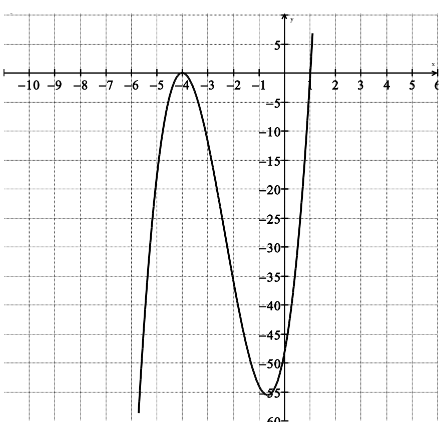
The same is true for polynomial functions, regardless of their degree.

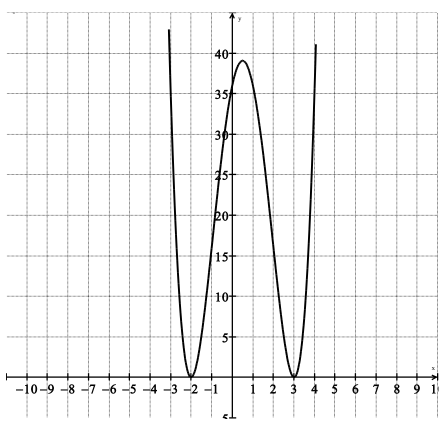
Eg. $y = \frac{1}{2}(x+2)(x-1)(x-5)$ has the x-ints of -2, 1, 5. Notice in the graph below that the graph passes through each x-intercept.

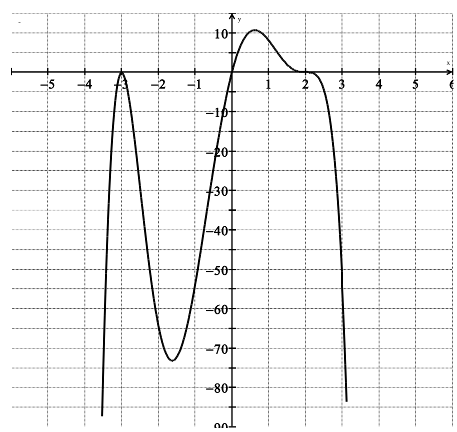


Look at the following graphs and their equations. What patterns occur at their x-intercepts?

$y = 3(x-1)(x+4)^2$ $\qquad$ $y = (x+2)^2(x-3)^2$ $\qquad$ $y = -\frac{1}{2}(x-2)^3(x+3)^2(x)$





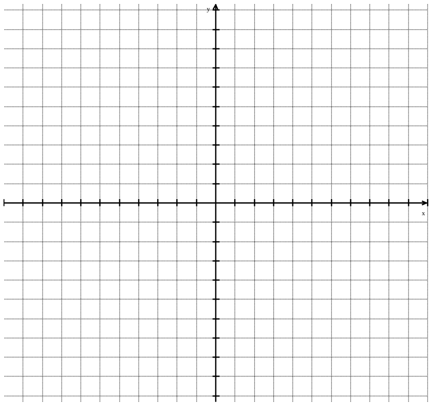


Note: The order of each root determines the shape of the graph at the root.

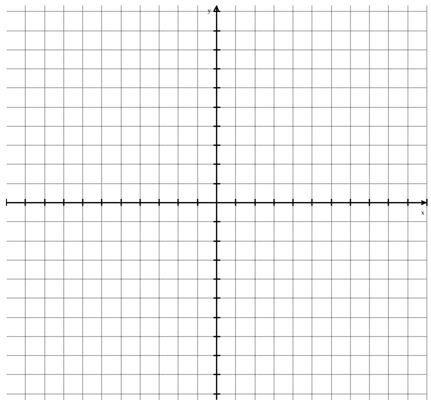
Order of Root	Shape of Graph at Root
Single	
Double	
Triple	

Example 1 Sketch each graph.

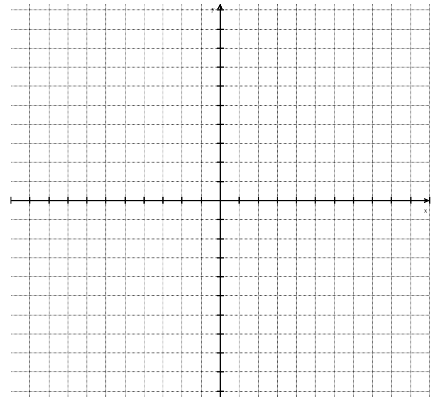
a) $y = (x + 2)(x - 5)^2$



b) $y = (x + 3)^2(x - 1)^2$

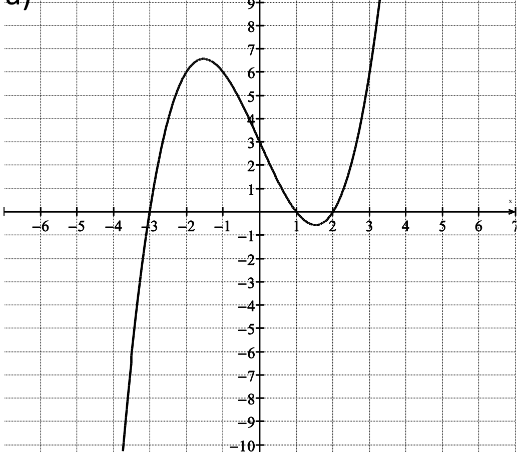


c) $y = -(x - 2)(x + 4)^3$

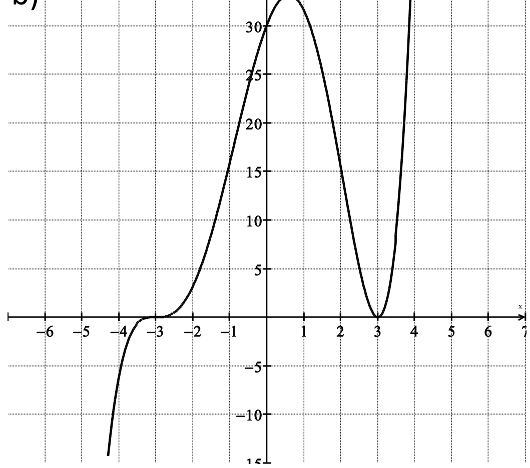


Example 2 Determine an equation for each of the following polynomial functions.

a)

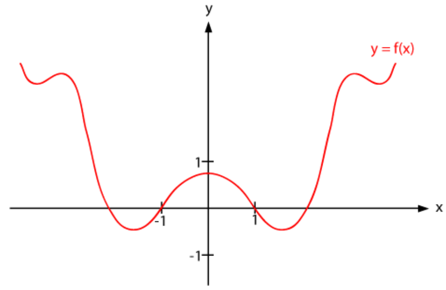


b)

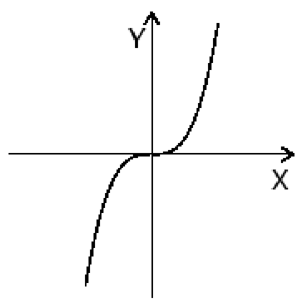


B. **<u>Odd/Even Functions</u>**

An **even function** satisfies the property that $\boldsymbol{f(-x) = f(x)}$ for all values of x in its domain. An even function is **symmetric about the y-axis**, which means it has a line of symmetry at x=0. **Not all even degree functions are even functions!**



An **odd function** satisfies the property that $\boldsymbol{f(-x) = -f(x)}$ for all values of x in its domain. Also, an even function is **point symmetric about the origin**, which means that (0,0) is a point of symmetry. **Not all odd degree functions are odd functions!**



Example Determine whether each function is odd, even, or neither.

a. $f(x) = x^4 - x^2 + 2$

b. $f(x) = x^3 + 3x$

c. $f(x) = x^3 + 2x^2 - 3$