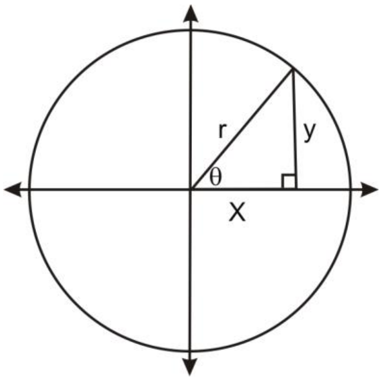
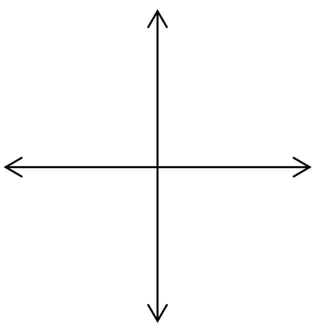
Finding Trigonometric Ratios

For any rotation angle, θ, in standard position that has a point P(x,y) on its terminal arm the primary trigonometric ratios for the angle can be expressed in terms of x, y, and r, where r is the distance from point P to the origin.



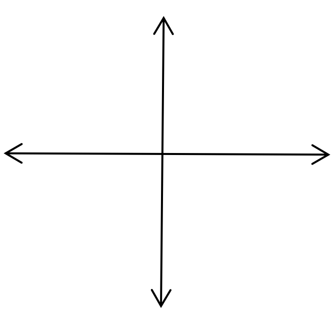
Example 1 P(-8,15) is a point on the terminal arm of angle α.

a. Calculate sin α, cos α, and tan α
b. Calculate the measure of angle α to the nearest degree.



Example 2 Angle β is in quadrant 4 and $\tan\beta = \frac{-3}{4}$.

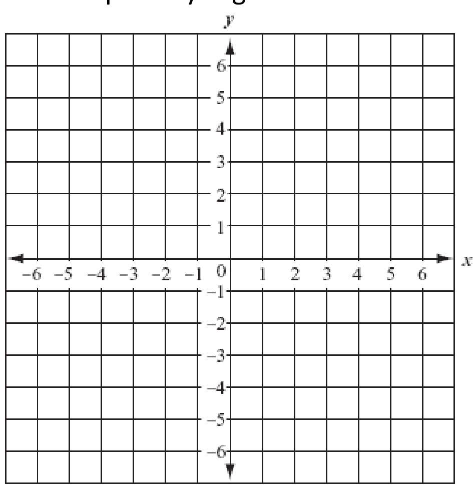
a. Find sin β and cos β.
b. Find the measure of angle β.



Example 3 Use the point P(0,1) to determine the values of sin 90°, cos 90°, and tan 90° without a calculator.

Example 4 Point F(5,3) is on the terminal arm of angle β.

a. Find the primary trigonometric ratios for angle β.



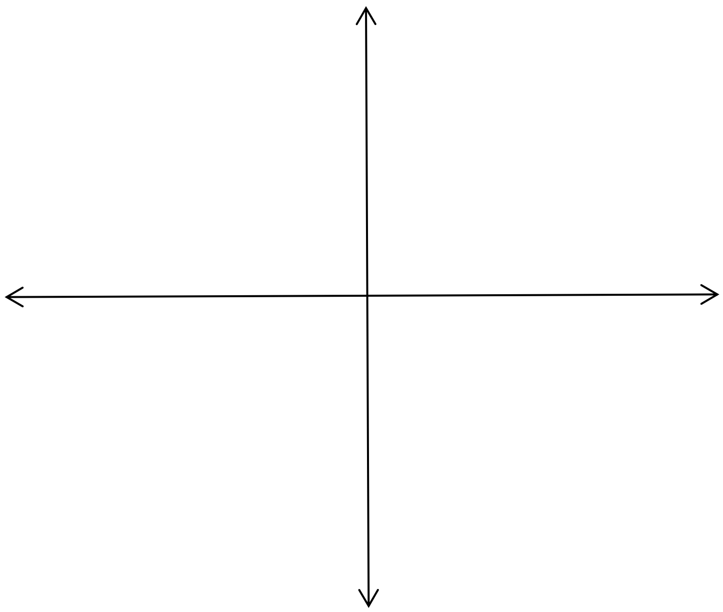
b. Determine the coordinates of the endpoints of the terminal arm for each angle.

i. 180° - β
ii. 180° + β
iii. 360° - β

c. Determine the primary trig ratios for each of the angles in part b. Complete the following table and compare the trig ratios for the four related angles.

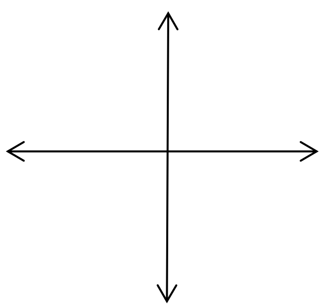
Angle	Sine	Cosine	Tangent

Notice that the Sine Cosine and Tangent Ratios are the same for the related angles; however, the sign changes depending on which quadrant the terminal arm lies in. This leads to what we call the CAST rule:



Example 5 Use special triangles and the CAST rule to find the exact value without a calculator.

a. sin 225°



b. cos 330°