Applications of Logarithms

A. Solving for Number of Time Periods in Exponential Growth and Decay Questions

Recall: The same equation can be used to describe both exponential growth and decay situations.

$$y = Ab^x$$

where: y is the amount after x time periods
A is the initial amount
b is the growth/decay factor (1 + growth rate/ 1 – decay rate)
x is the number of time periods

Example 1 An investment of $2000 earns 4.2% interest, compounded annually.

a. Write an equation for A, the amount of the investment, after t years.

b. How long will it take for the investment to double in value?

B. pH Scale

The pH of a substance is a measure if its acidity. $pH = - \log [H^+]$, where H^+ is the hydrogen ion concentration in a substance, measured in moles per litre.

Example Find the concentration of hydrogen ions in a substance that has a pH of 5.

C. *Decibel Scale*

The difference in sound levels, in decibels, can be found using the equation

$\beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right)$, where $\beta_2 - \beta_1$ is the difference in sound levels, in decibels, and $\frac{I_2}{I_1}$ is the ratio of their sound intensities, where I is measured in watts per square metre (W/m^2).

Example The sound level of a whisper is 30 dB and the sound level of normal conversation is 60 dB. How many times as intense as a whisper is the sound of a normal conversation?

Example The sound level in normal city traffic is approximately 85 dB. The sound level while riding a snowmobile is about 32 times as intense. What is the sound level while riding a snowmobile, in decibels?

D. Richter Scale

The magnitude, *M,* of an earthquake is measured using the Richter Scale, which is defined as $M = log\left(\frac{I}{I_0}\right)$, where *I* is the intensity if the earthquake being measured and I_0 is the intensity of a standard, low-level earthquake.

Example How many times as intense as a standard earthquake is an earthquake measuring 2.4 on the Richter scale?

Example What is the magnitude of an earthquake 1000 times as intense as a standard earthquake?